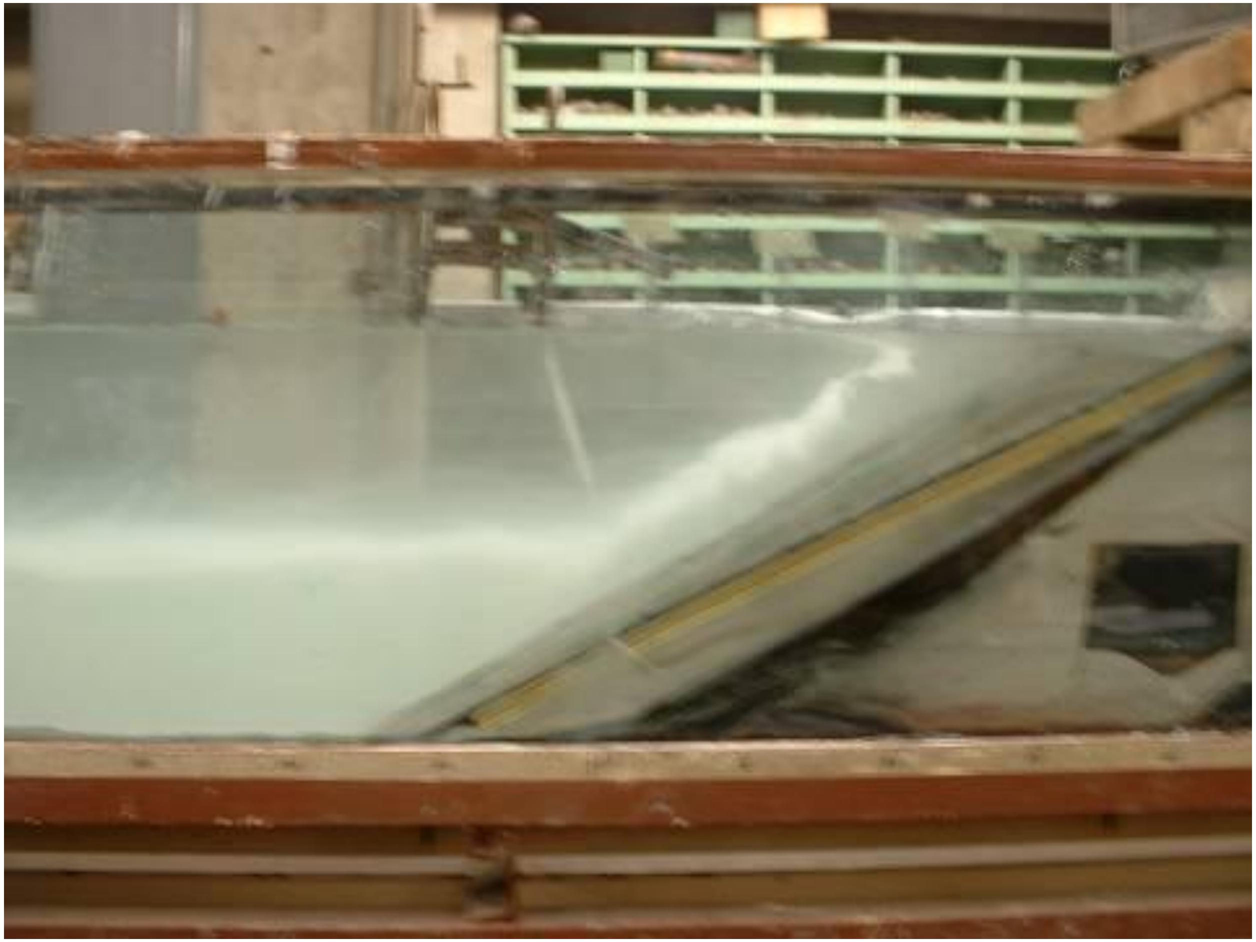
HYDRAULIC JUMPS



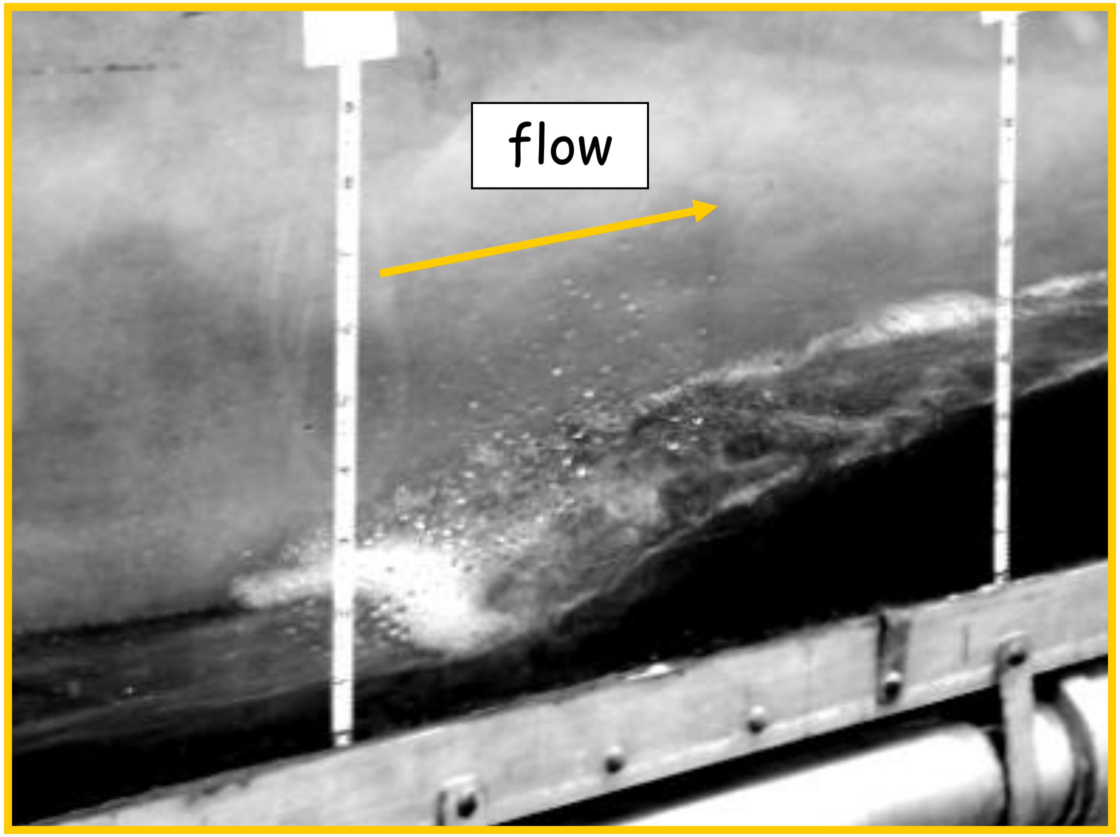
Hydraulic jump of a turbidity current in the laboratory. Flow is from right to left.

WHAT IS A HYDRAULIC JUMP?

A hydraulic jump is a type of *shock*, where the flow undergoes a sudden transition from swift, thin (shallow) flow to tranquil, thick (deep) flow.

Hydraulic jumps are most familiar in the context of open-channel flows.

The image shows a hydraulic jump in a laboratory flume.

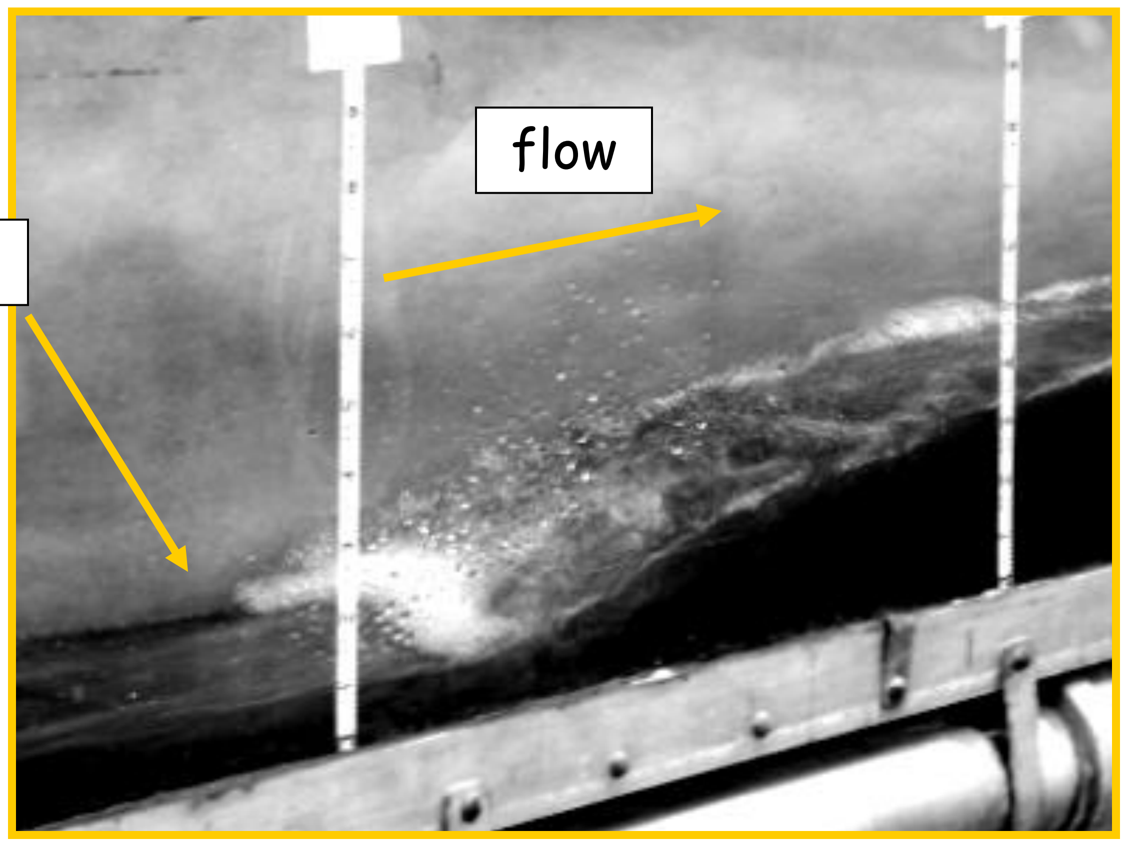


THE CHARACTERISTICS OF HYDRAULIC JUMPS

Hydraulic jumps in open-channel flow are characterized a drop in Froude number **Fr**, where

$$\mathbf{Fr} = \frac{U}{\sqrt{gH}}$$

from supercritical (**Fr** > 1) to subcritical (**Fr** < 1) conditions. The result is a step increase in depth H and a step decrease in flow velocity U passing through the jump.



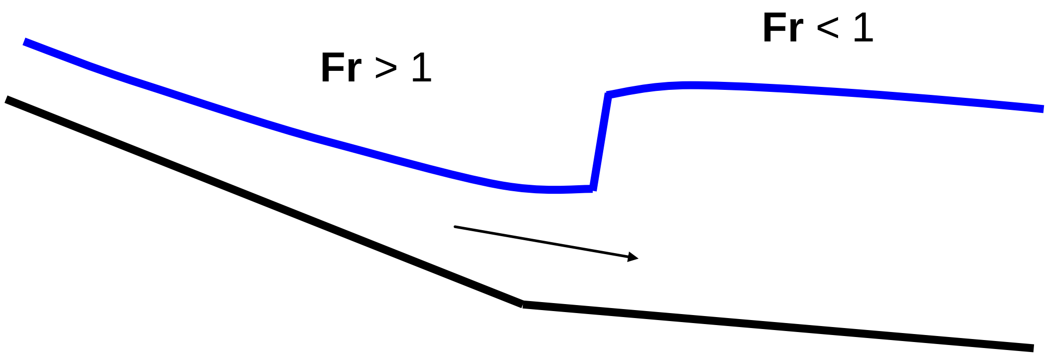
WHAT CAUSES HYDRAULIC JUMPS?

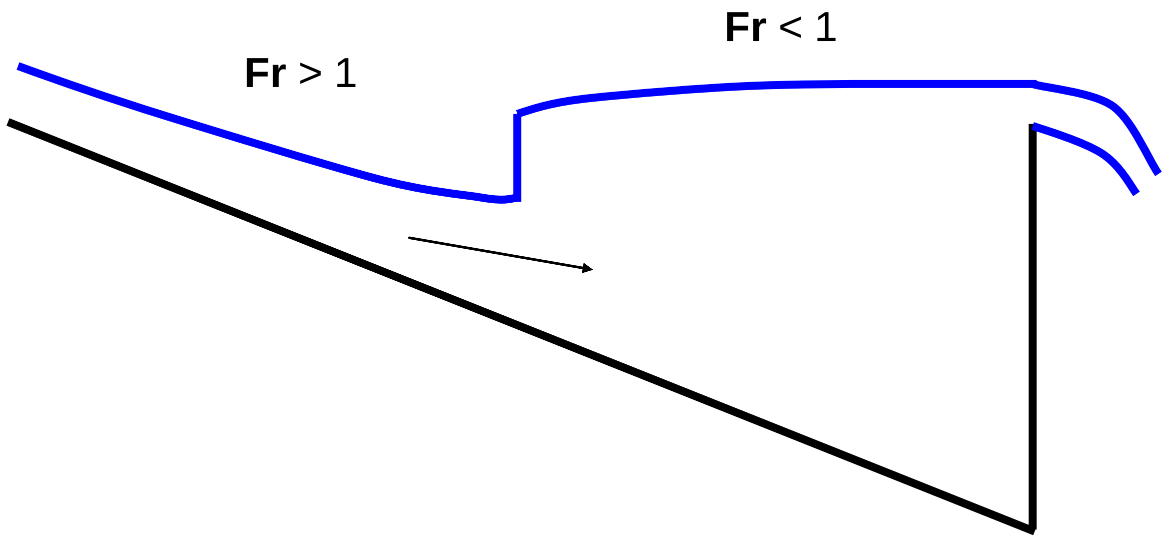
The conditions for a hydraulic jump can be met where

a) the upstream flow is supercritical, and

b) slope suddenly or gradually decreases downstream, or

c) the supercritical flow enters a confined basin.





INTERNAL HYDRAULIC JUMPS

Hydraulic jumps in rivers are associated with an extreme example of flow stratification: flowing water under ambient air.

Internal hydraulic jumps form when a denser, fluid flows under a lighter ambient fluid. The photo shows a hydraulic jump as relatively dense air flows east across the Sierra Nevada Mountains, California.

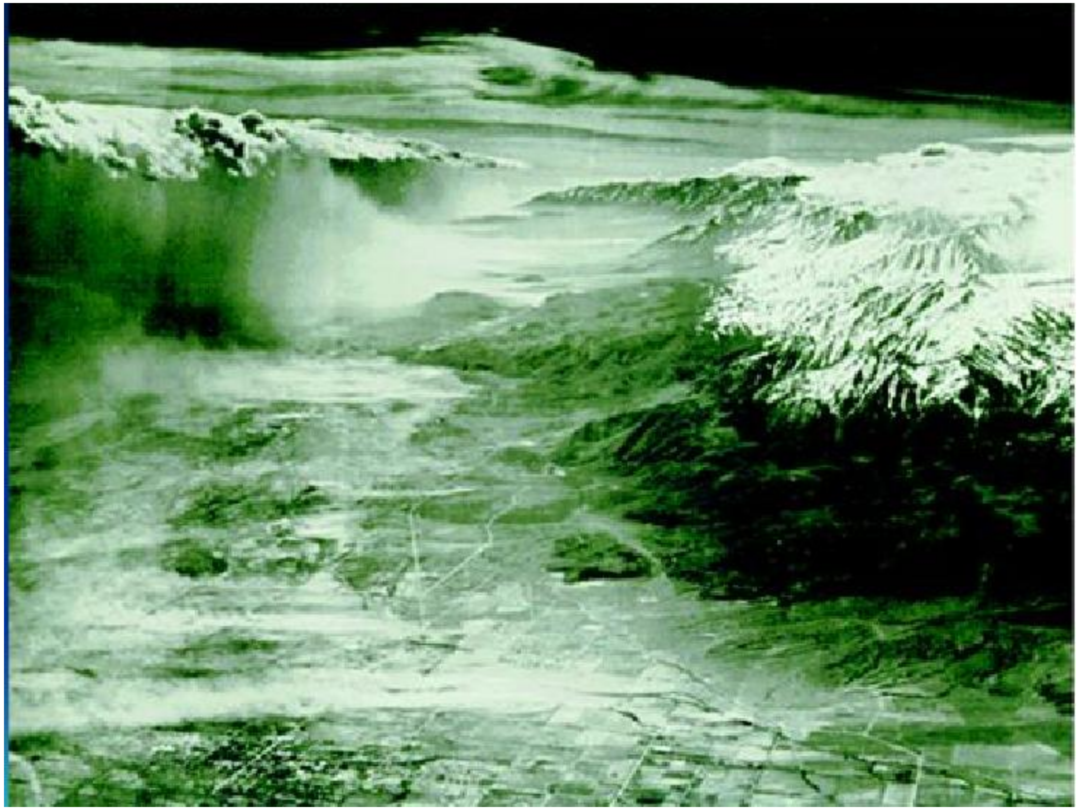


Photo by Robert Symons, USAF, from the Sierra Wave Project in the 1950s.

DENSIMETRIC FROUDE NUMBER

Internal hydraulic jumps are mediated by the *densimetric Froude number* $\mathbf{Fr}_d$, which is defined as follows for a turbidity current.

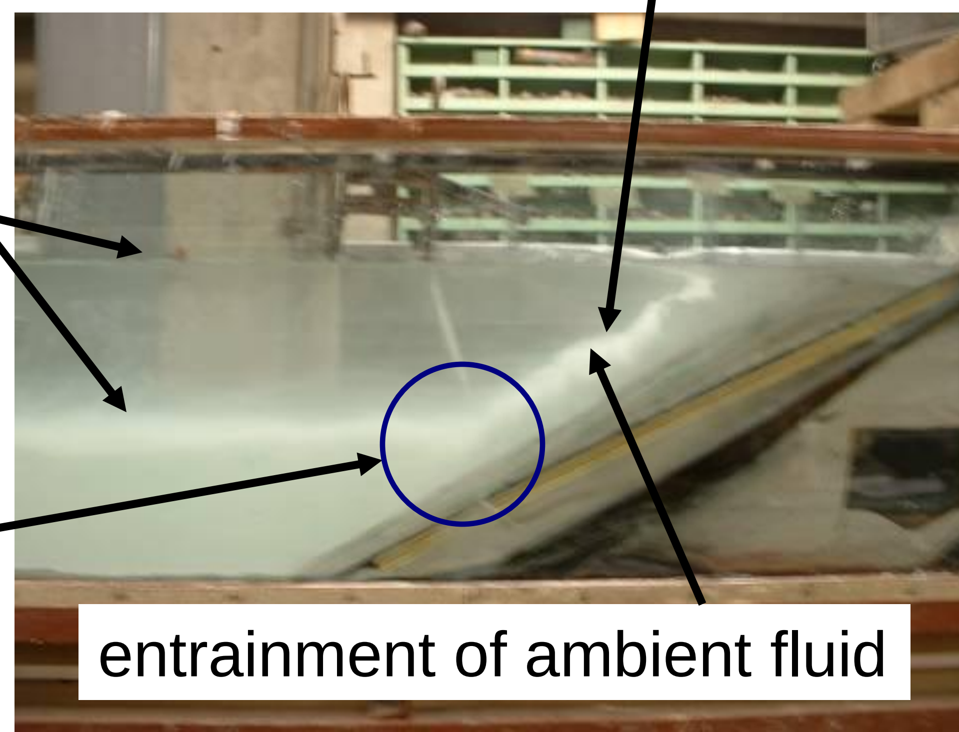
$$\mathbf{Fr}_d = \frac{U}{\sqrt{RCgH}}$$

U = flow velocity
g = gravitational acceleration
H = flow thickness
C = volume suspended sediment concentration
$R = \rho_s/\rho - 1 \cong 1.65$

Subcritical: $\mathbf{Fr}_d < 1$ ← *Supercritical:* $\mathbf{Fr}_d > 1$

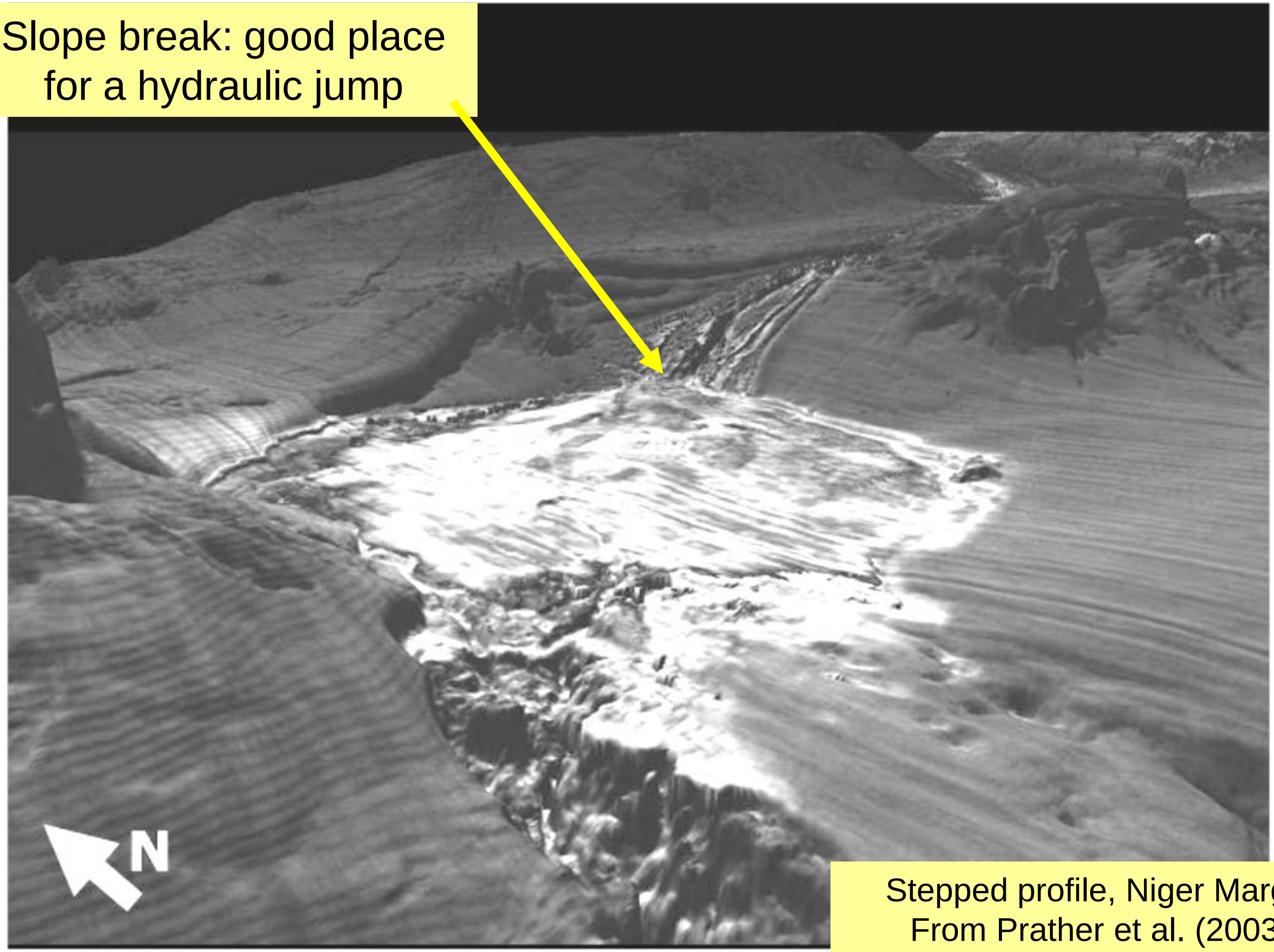
Water surface

internal hydraulic jump



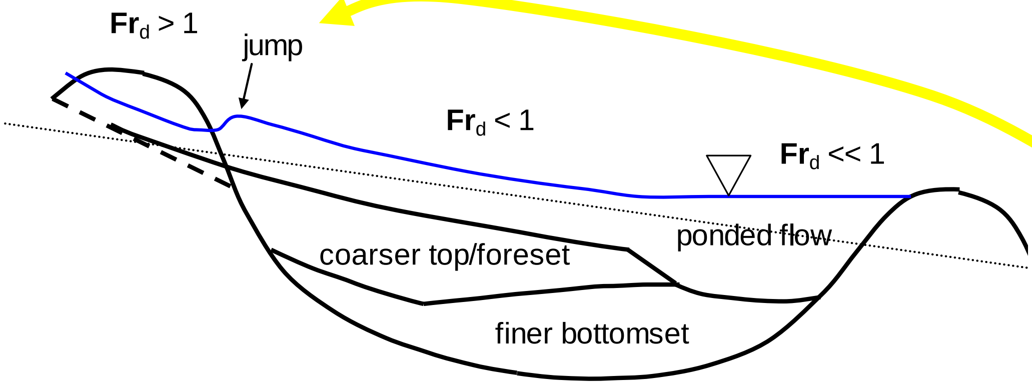
INTERNAL HYDRAULIC JUMPS AND TURBIDITY CURRENTS

Slope break: good place for a hydraulic jump

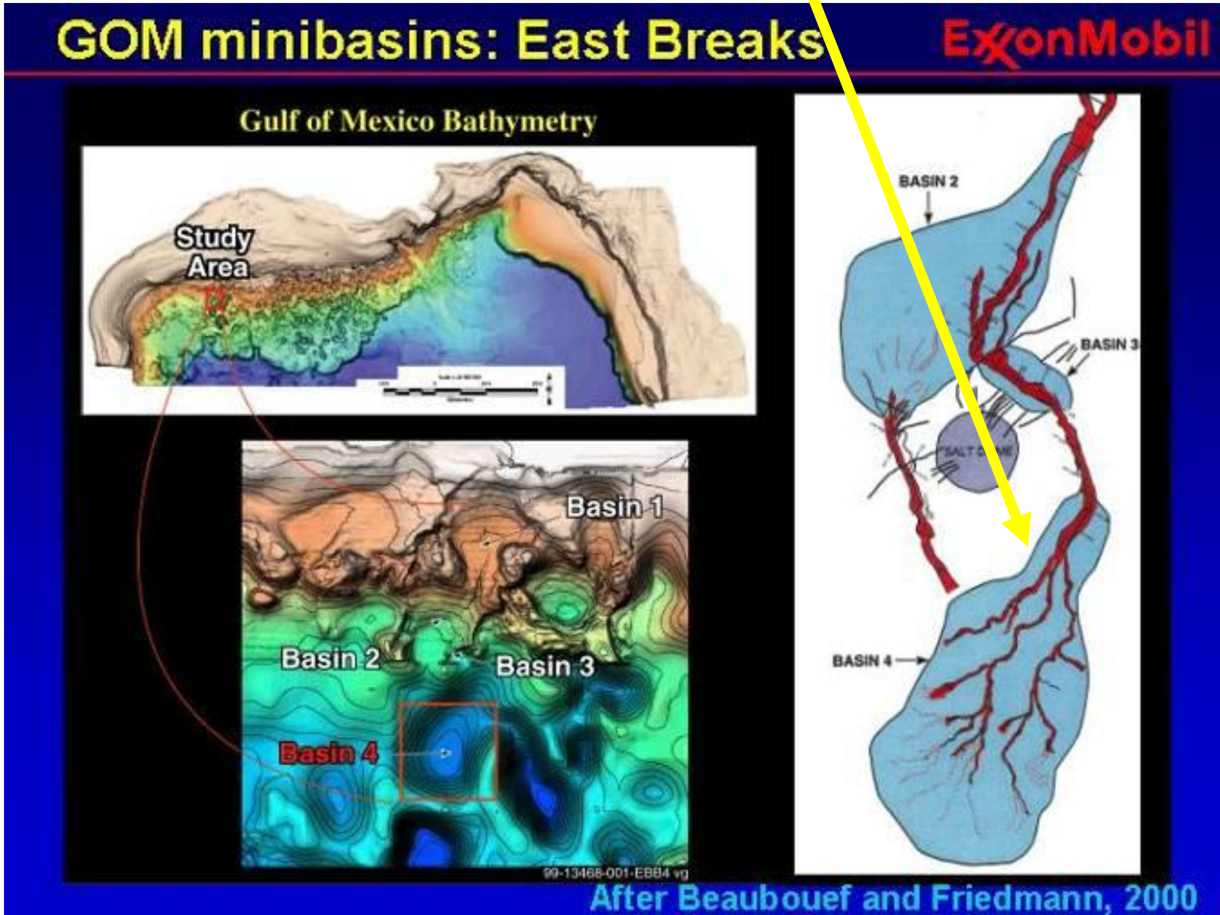


Stepped profile, Niger Margin
From Prather et al. (2003)

INTERNAL HYDRAULIC JUMPS AND TURBIDITY CURRENTS



Flow into a confined basin: good place for a hydraulic jump



ANALYSIS OF THE INTERNAL HYDRAULIC JUMP

Definitions: "u" → upstream and "d" → downstream

U = flow velocity

C = volume suspended sediment concentration

z = upward vertical coordinate

p = pressure

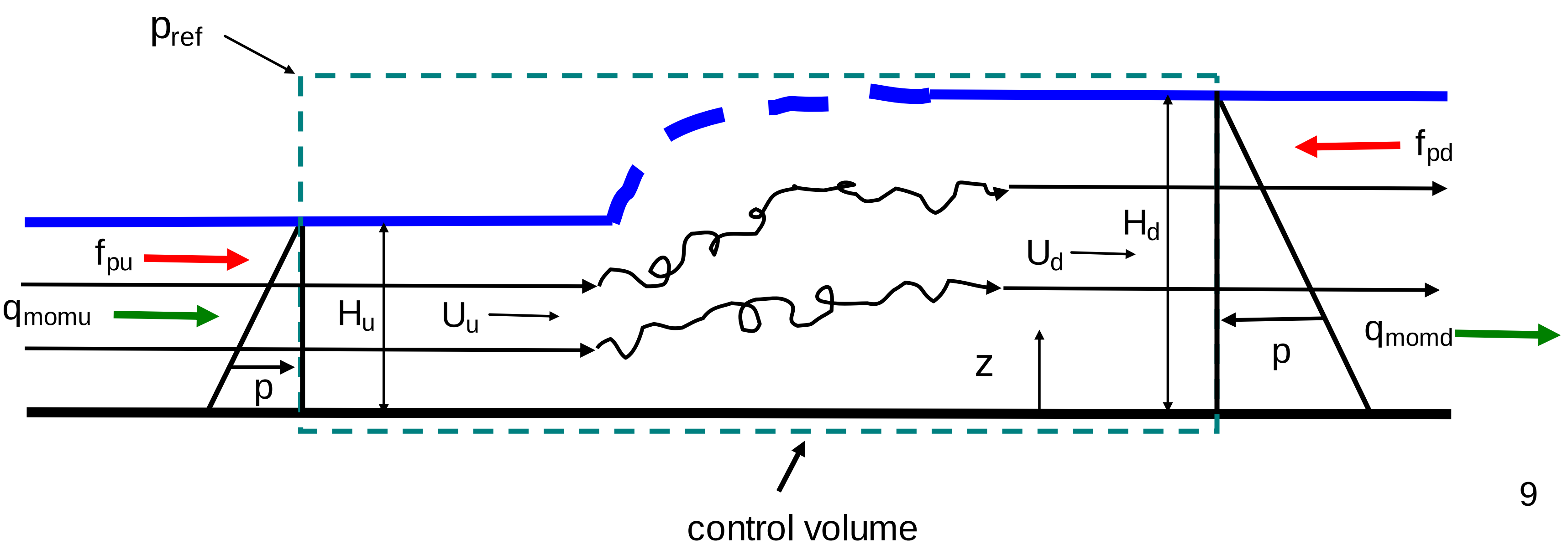
p_{ref} = pressure force at z = H_d (just above turbidity current

f_p = pressure force per unit width

q_{mom} = momentum discharge per unit width

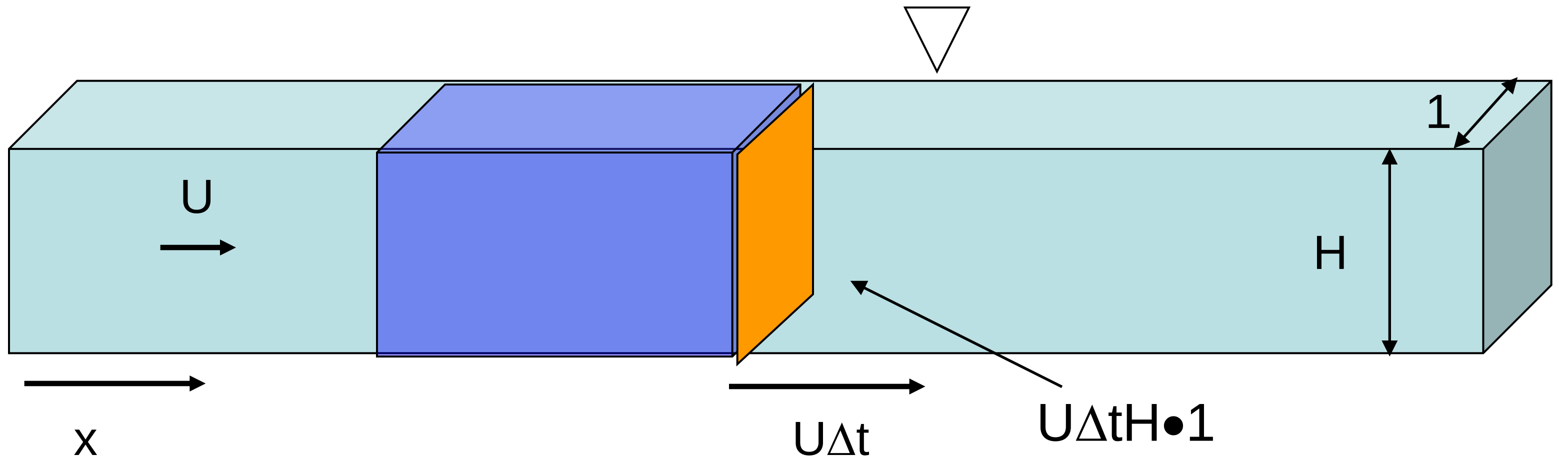
Flow in the control volume is steady.

USE TOPHAT ASSUMPTIONS FOR U AND C.



VOLUME, MASS, MOMENTUM DISCHARGE

H = depth
U = flow velocity
Channel has a unit width 1

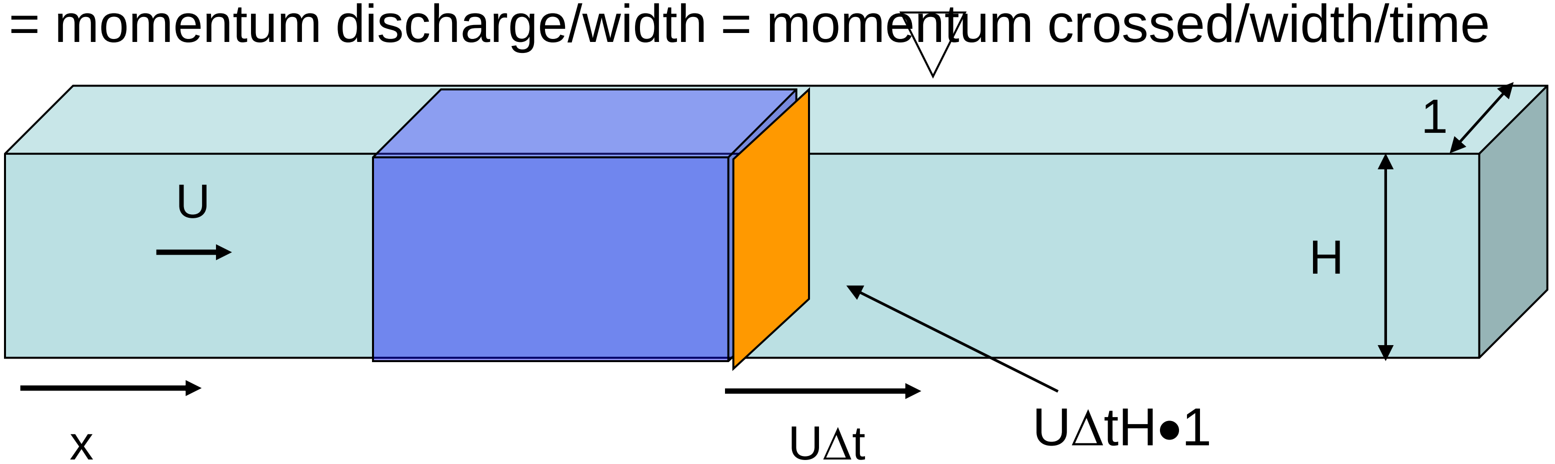


In time Δt a fluid particle flows a distance UΔt

The **volume** that crosses normal to the section in time Δt = UΔtH•1
The **flow mass** that crosses normal to the section in time Δt is density x volume crossed = $\rho(1+RC)U\Delta tH \bullet 1 \cong \rho U \Delta t H$
The **sediment mass** that crosses = $\rho_s C U \Delta t H \bullet 1$
The **momentum** that cross normal to the section is mass x velocity = $\rho(1+RC)U\Delta tH \bullet 1 \bullet U \cong \rho U^2 \Delta t H$

VOLUME, MASS, MOMENTUM DISCHARGE (contd.)

q_f = volume discharge per unit width = volume crossed/width/time
q_{mass} = flow mass discharge per unit width = mass crossed/width/time
$q_{sedmass}$ = sediment mass discharge per unit with = mass crossed/width/time
q_{mom} = momentum discharge/width = momentum crossed/width/time



$q_f = U\Delta tH\bullet 1/(\Delta t\bullet 1)$ thus $q_f = UH$

$q_{mass} = \rho U\Delta tH\bullet 1/(\Delta t\bullet 1)$ thus $q_{mass} = \rho UH$

$q_{sedmass} = \rho_s CU\Delta tH\bullet 1/(\Delta t\bullet 1)$ thus $q_{sedmass} = \rho_s CUH$

$q_{mom} = \rho U\Delta tH\bullet 1\bullet U/(\Delta t\bullet 1)$ thus $q_{mom} = \rho U^2H$

FLOW MASS BALANCE ON THE CONTROL VOLUME

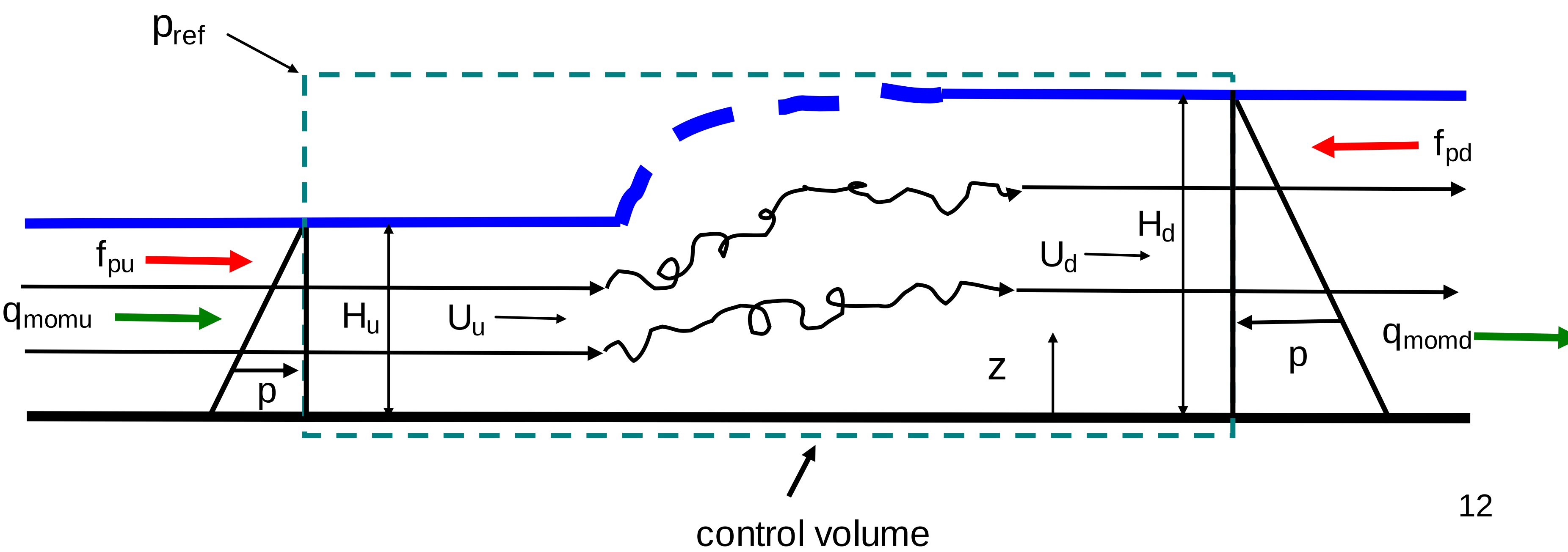
∂/∂t(fluid mass in control volume) = net mass inflow rate

$$0 = q_{massu} - q_{massd} \quad \therefore \quad q_{mass} = const$$

or

$$0 = \rho U_u H_u - \rho U_d H_d \quad \therefore \quad q_{mass} = constant = \rho q_f$$

$$\text{where} \quad q_f = UH = \text{flow discharge / width}$$

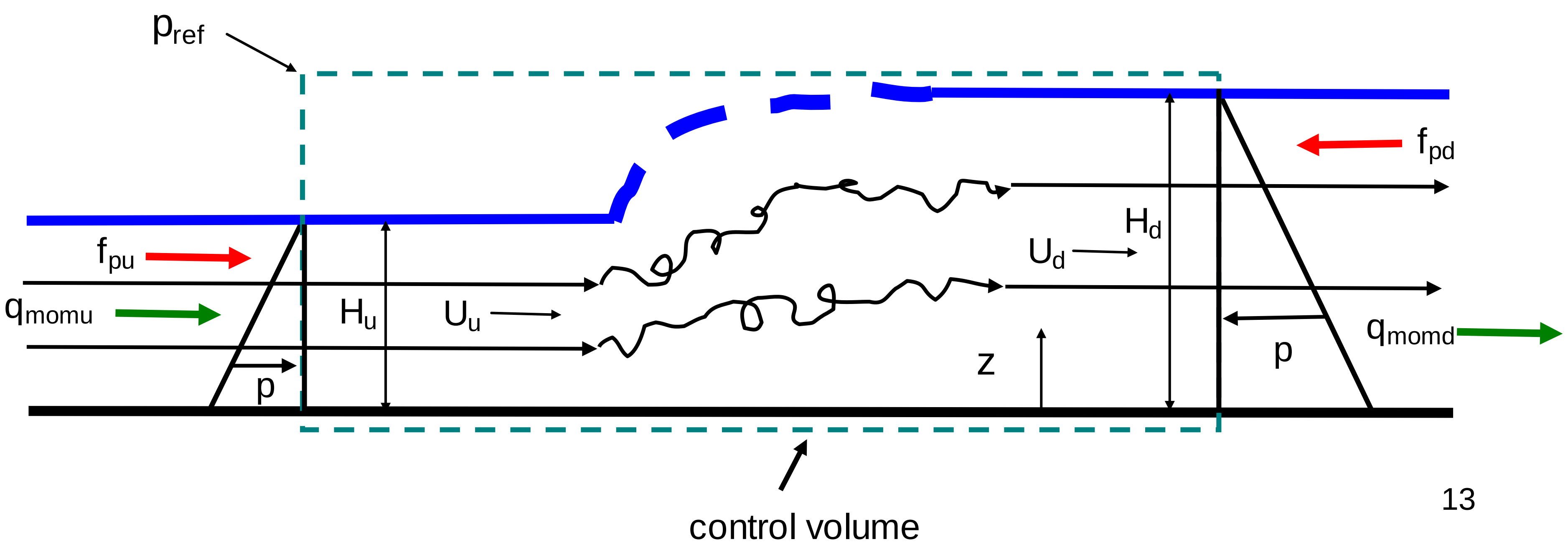


FLOW MASS BALANCE ON THE CONTROL VOLUME contd/

Thus flow discharge

$$q_f = UH$$

is constant across the hydraulic jump



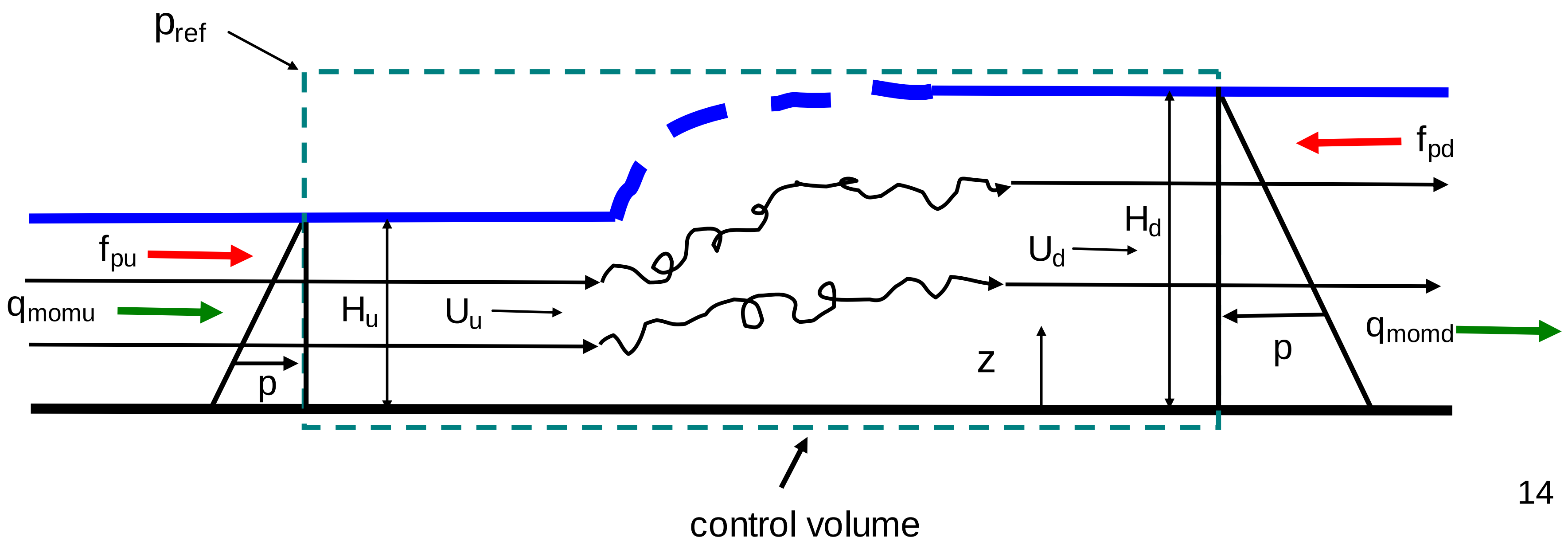
BALANCE OF SUSPENDED SEDIMENT MASS ON THE CONTROL VOLUME

∂/∂t(sediment mass in control volume) = net sediment mass inflow rate

$$0 = q_{sedmassu} - q_{sedmassd} \quad \therefore \quad q_{sedmass} = const$$

or

$$0 = \rho_s C_u U_u H_u - \rho_s C_d U_d H_d \quad \therefore \quad q_{sedmass} = constant$$



BALANCE OF SUSPENDED SEDIMENT MASS ON THE CONTROL VOLUME contd

Thus if the volume sediment discharge/width is defined as

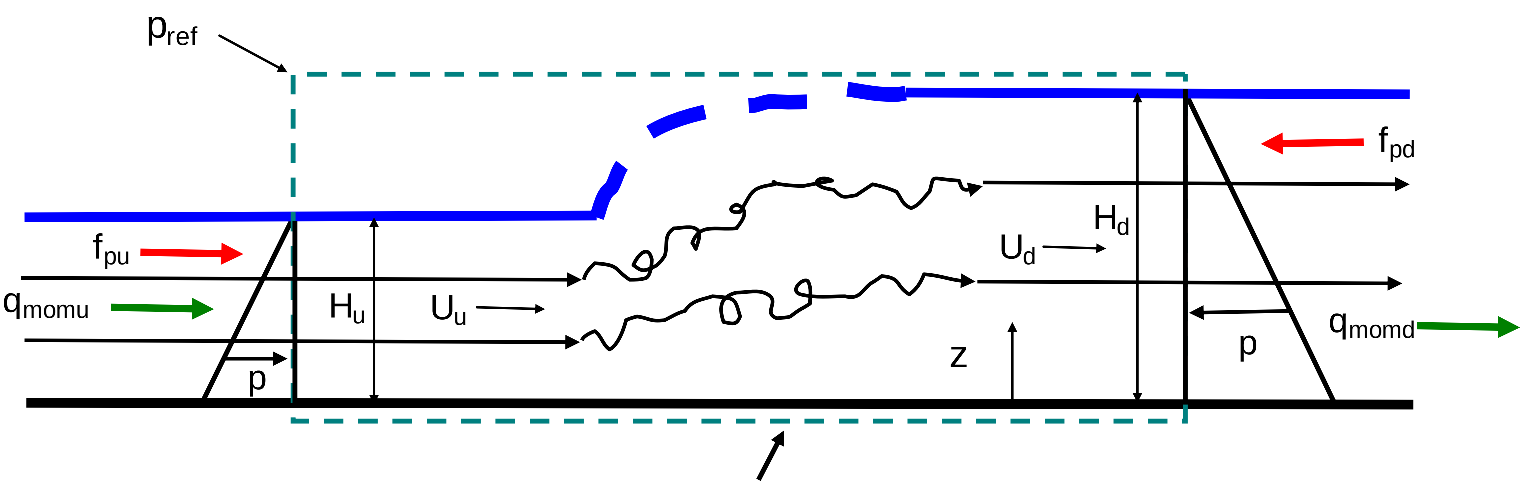
$$q_{sedvol} = CUH$$

then $q_{sedvol} = q_{sedmass}/\rho_s$ is constant across the jump.

But if

$$q_f = UH = const \quad , \quad q_{sedvol} = CUH = const$$

then **C is constant across the jump!**



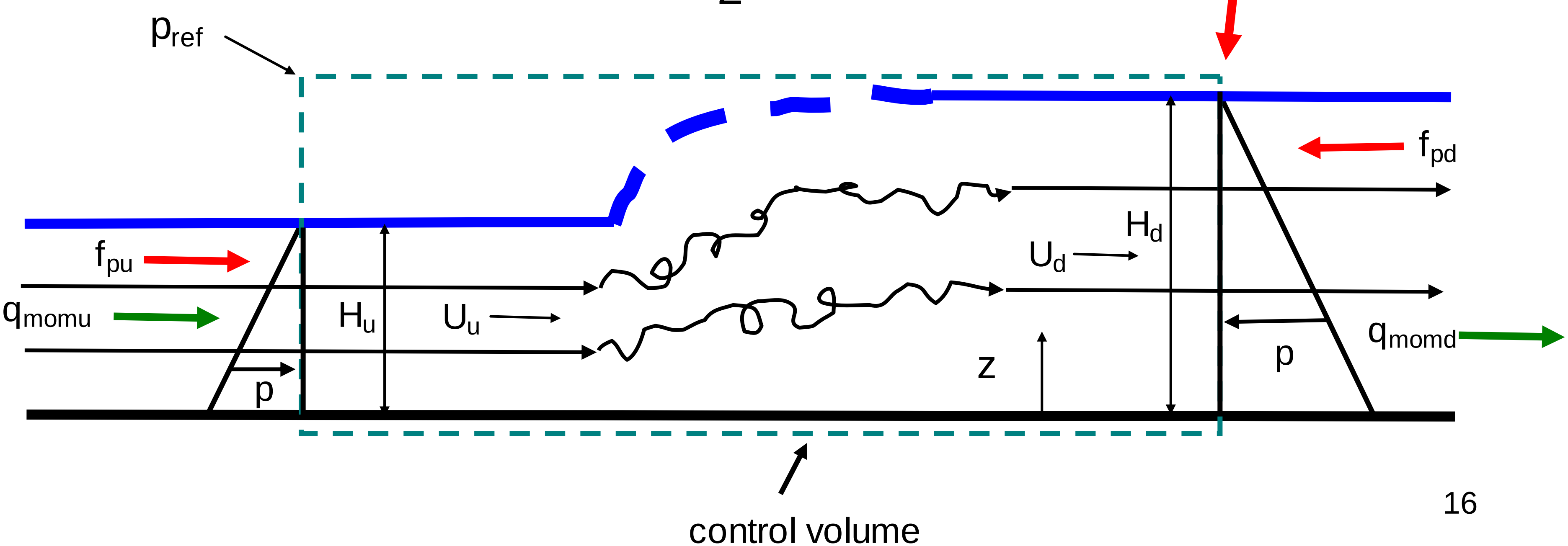
PRESSURE FORCE/WIDTH ON DOWNSTREAM SIDE OF CONTROL VOLUME

$$\frac{dp}{dz} = -\rho g(1+RC) \quad , \quad p\big|_{z=H_d} = p_{ref}$$

$$\therefore$$

$$p = p_{ref} + \rho g(1+RC)(H_d - z)$$

$$f_{pd} = \int_0^{H_d} p dz \cdot 1 = \frac{1}{2}\rho g(1+RC)H_d^2$$

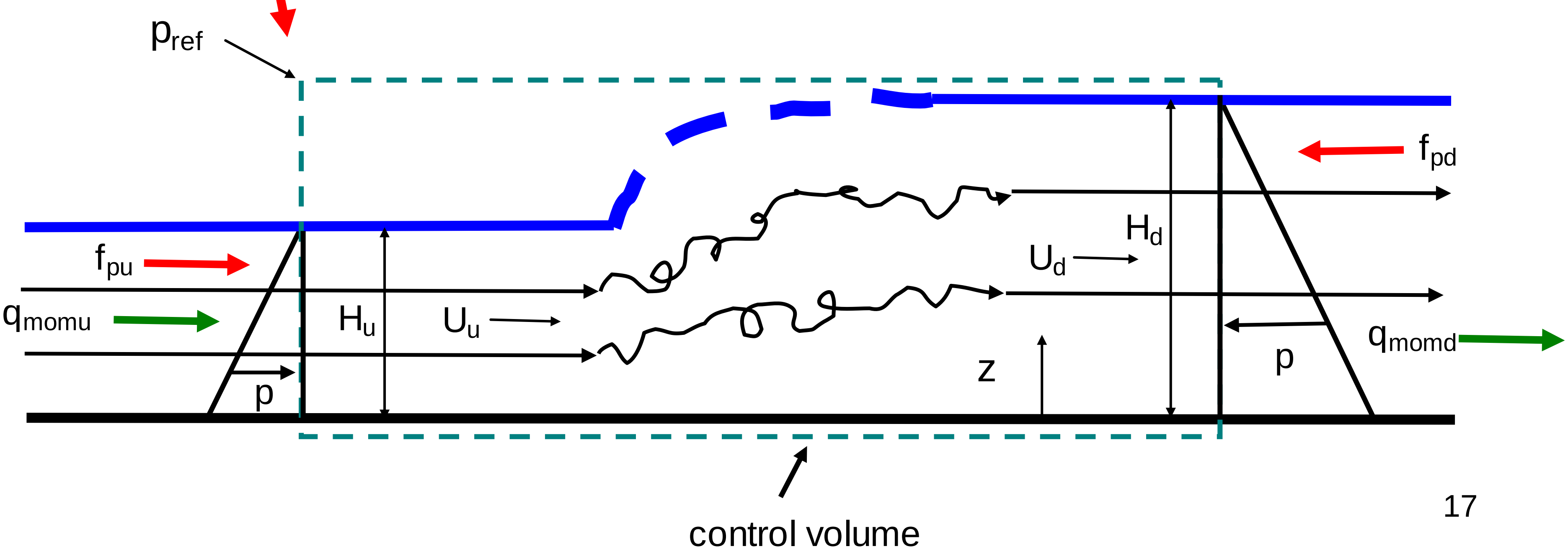


PRESSURE FORCE/WIDTH ON UPSTREAM SIDE OF CONTROL VOLUME

$$\frac{dp}{dz} = \begin{cases} -\rho g & , \quad H_u < z < H_d \\ -\rho g(1+RC) & , \quad 0 < z \le H_u \end{cases} \quad , \quad p\big|_{z=H_d} = p_{ref}$$

$$\therefore$$

$$p = \begin{cases} p_{ref} + \rho g(H_d - z) & , \quad H_u < z < H_d \\ p_{ref} + \rho g(H_d - H_u) + \rho g(1+RC)(H_u - z) & , 0 < z < H_u \end{cases}$$



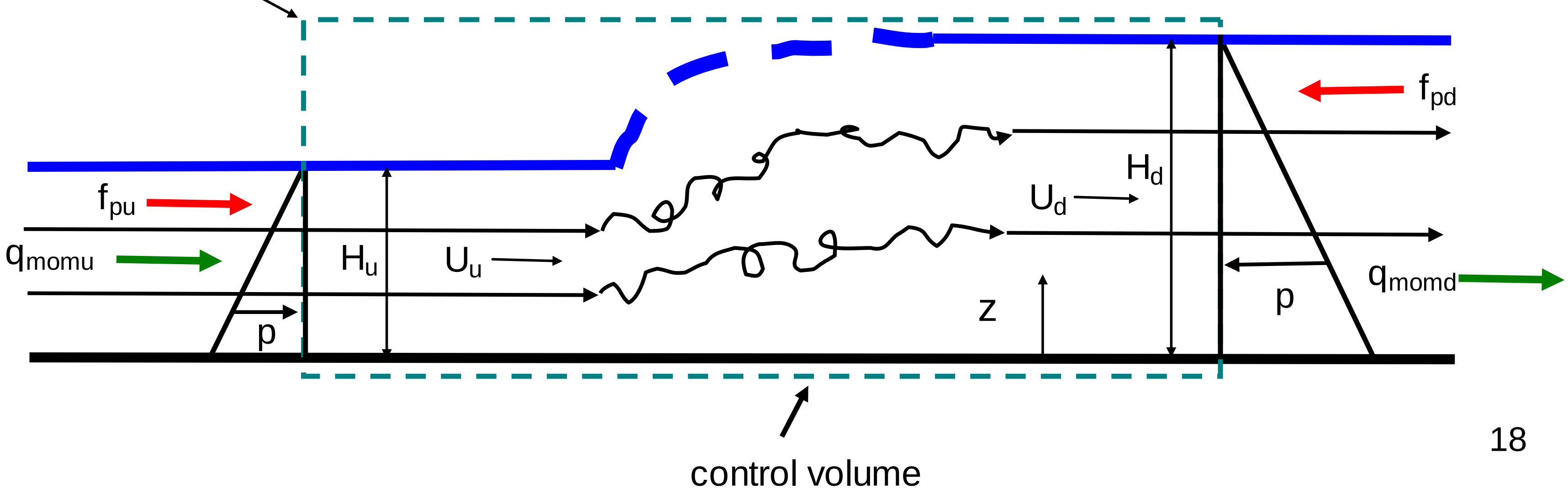
PRESSURE FORCE/WIDTH ON UPSTREAM SIDE OF CONTROL VOLUME contd.

$$p = \begin{cases} p_{ref} + \rho g(H_d - z) & , \quad H_u < z < H_d \\ p_{ref} + \rho g(H_d - H_u) + \rho g(1+RC)(H_u - z) & , 0 < z < H_u \end{cases}$$

$$f_{pu} = \int_0^{H_d} p dz \cdot 1 = \int_0^{H_u} p dz \cdot 1 + \int_{H_u}^{H_d} p dz \cdot 1 =$$

$$p_{ref} H_u + \rho g(H_d - H_u)H_u + \frac{1}{2}\rho(1+RC)H_u^2 + p_{ref}(H_d - H_u) + \frac{1}{2}\rho g(H_d - H_u)^2$$

$$= p_{ref} H_d + \frac{1}{2}\rho(1+RC)H_u^2 + \rho g(H_d - H_u)H_u + \frac{1}{2}\rho g(H_d - H_u)^2$$

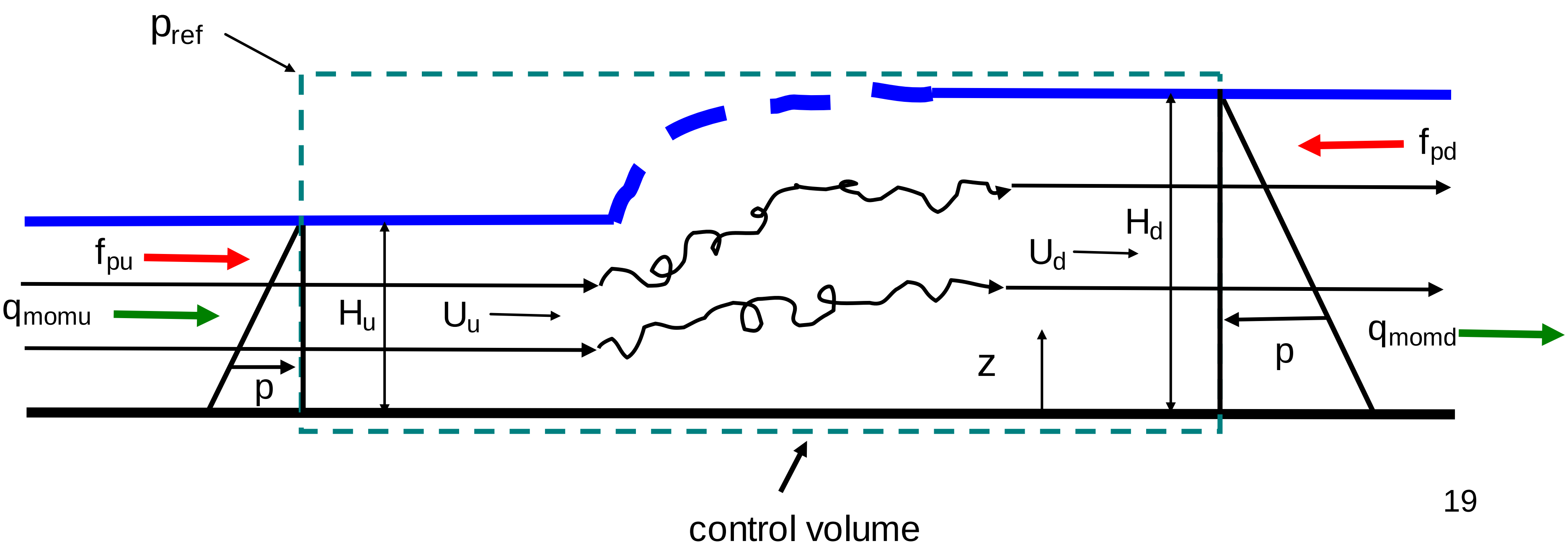


NET PRESSURE FORCE

$$f_{pnet} = f_{pu} - f_{pd} = p_{ref}H_d + \frac{1}{2}\rho(1+RC)H_u^2 + \rho g(H_d - H_u)H_u + \frac{1}{2}\rho g(H_d - H_u)^2$$

$$- p_{ref}H_d - \frac{1}{2}\rho(1+RC)gH_d^2 =$$

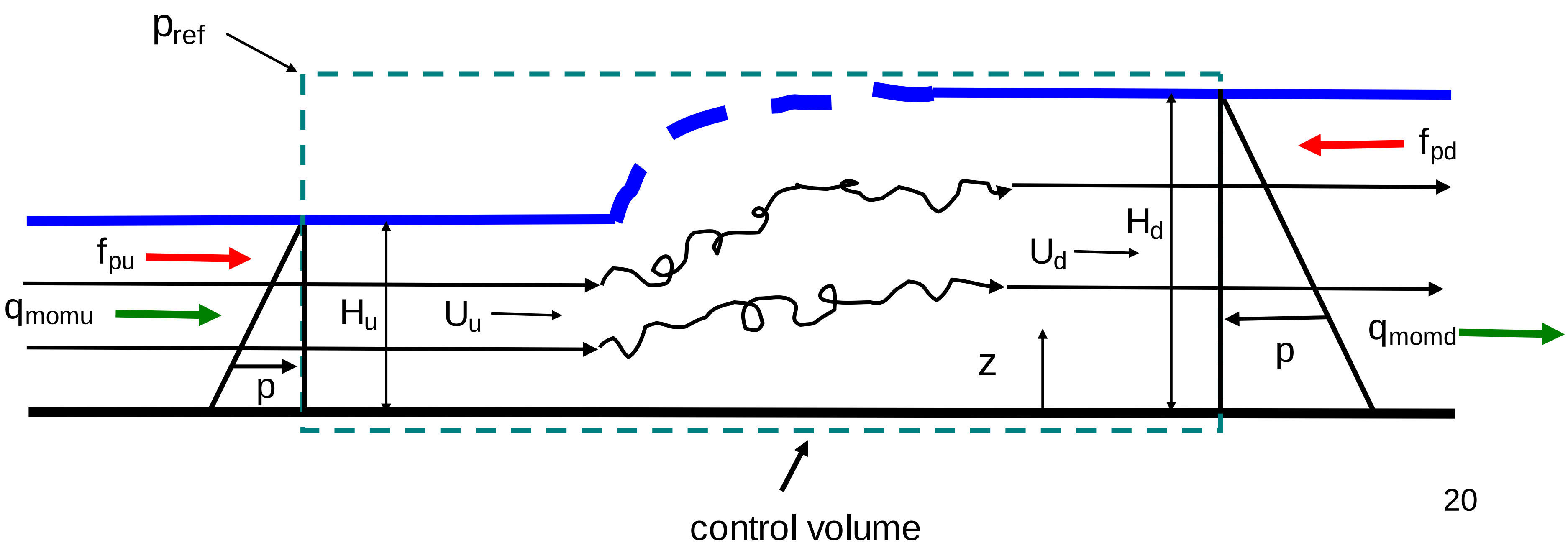
$$\frac{1}{2}\rho RCg\left(H_u^2 - H_d^2\right)$$



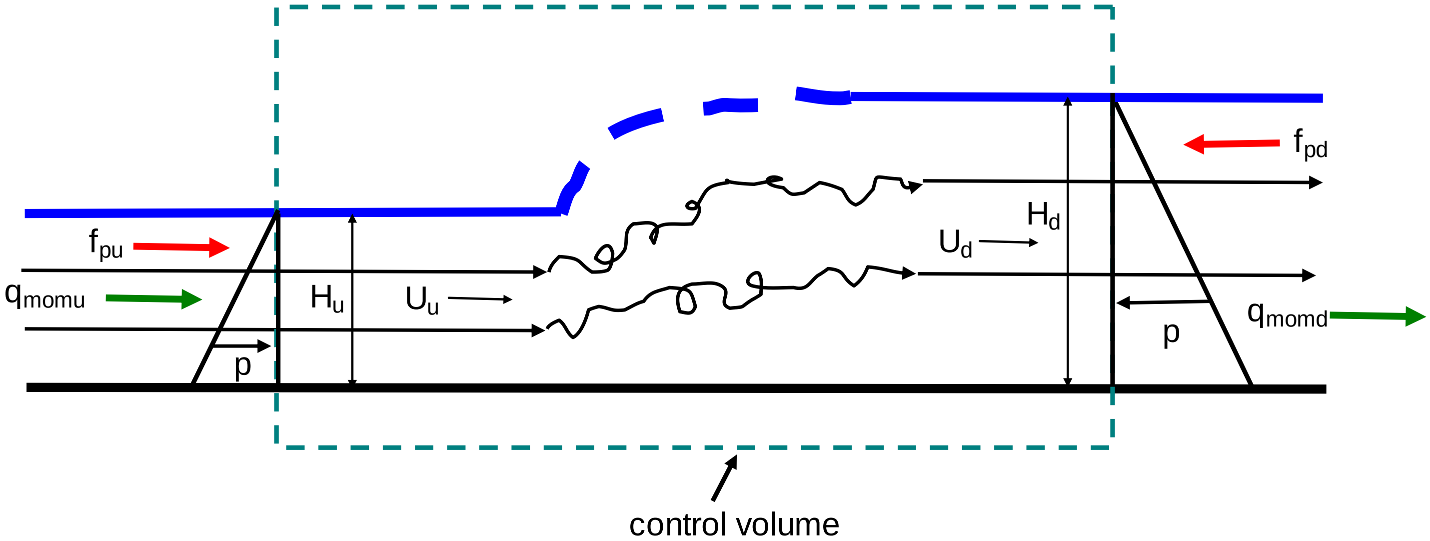
STREAMWISE MOMENTUM BALANCE ON CONTROL VOLUME

∂/∂(momentum in control volume) = Σforces + net inflow rate of momentum

$$0 = \frac{1}{2}\rho RCgH_u^2 - \frac{1}{2}\rho RCgH_d^2 + \rho U_u^2 H_u - \rho U_d^2 H_d$$



REDUCTION



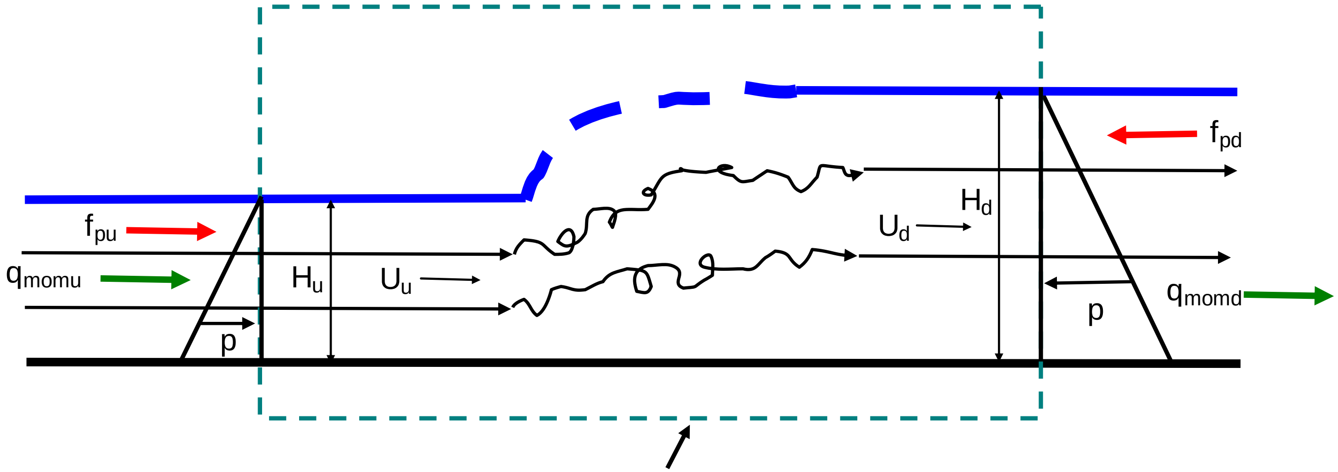
$$UH = q_f \quad \therefore \quad U^2H = Uq_f = \frac{q_f^2}{H}$$

thus

$$0 = \frac{1}{2}RCgH_u^2 - \frac{1}{2}RCgH_d^2 + U_u^2H_u - U_d^2H_d \rightarrow$$

$$0 = \frac{1}{2}RCgH_u^2 - \frac{1}{2}RCgH_d^2 + \frac{q_f^2}{H_u} - \frac{q_f^2}{H_d}$$

REDUCTION (contd.)



$$0 = \frac{1}{2}RCgH_u^2 - \frac{1}{2}RCgH_d^2 + \frac{q_w^2}{H_u} - \frac{q_w^2}{H_d}$$

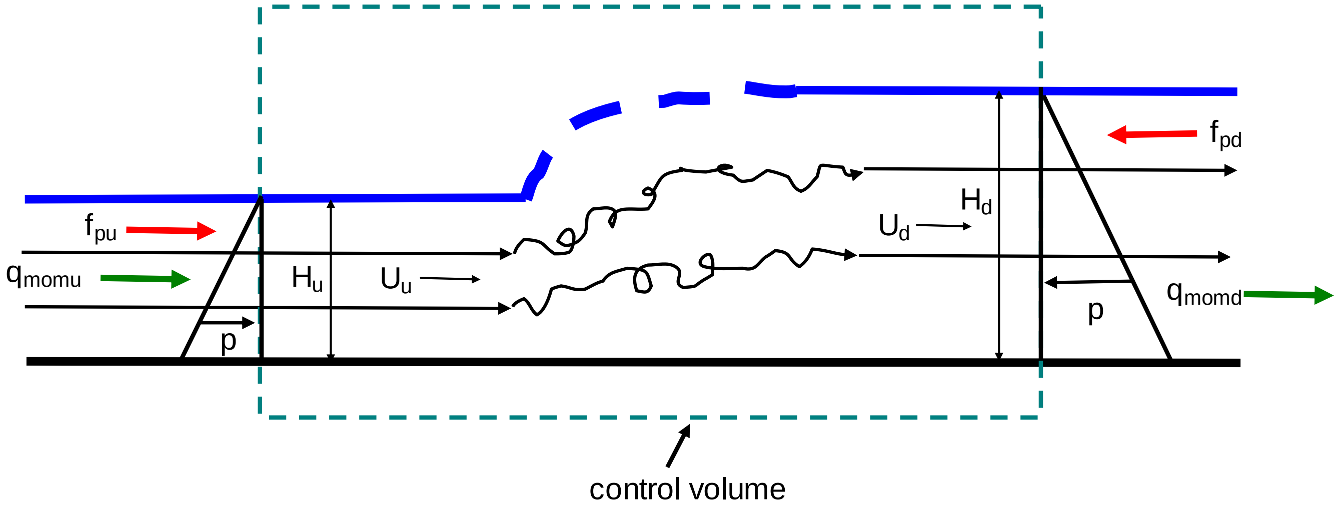
Now define $\phi = H_d/H_u$ (we expect that $\phi \geq 1$). Also

$$\mathbf{Fr}_{du} = \frac{U_u}{\sqrt{RCgH_u}} = \frac{q_f}{\sqrt{RCg}\,H_u^{3/2}}$$

Thus

$$2\mathbf{Fr}_{du}^2\left(1 - \frac{1}{\phi}\right) + 1 - \phi^2 = 0$$

REDUCTION (contd.)

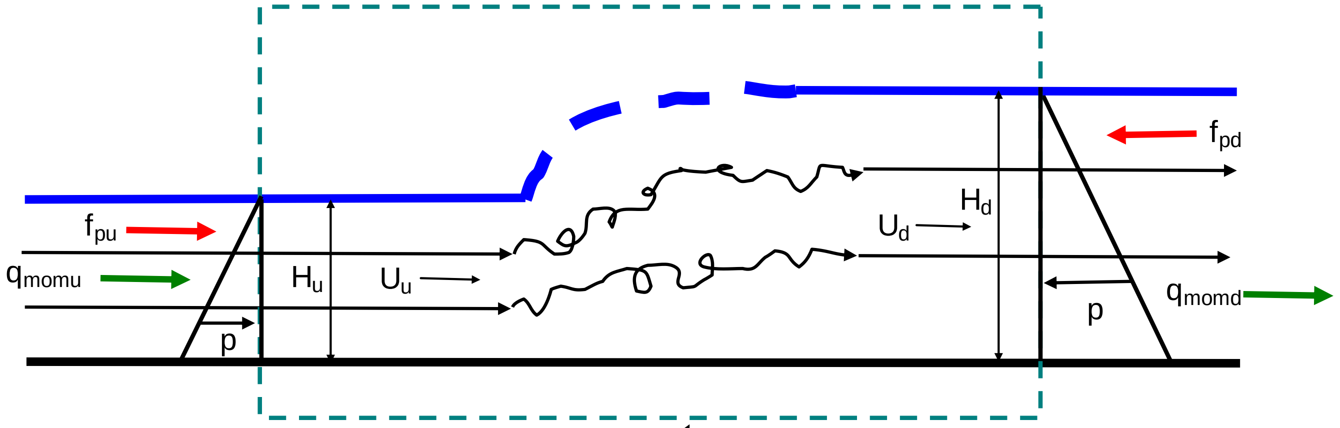


But $\left(1-\frac{1}{\phi}\right)=\left(\frac{\phi-1}{\phi}\right)$ $\quad 1-\phi^2=-(\phi+1)(\phi-1)$

$$\mathbf{2Fr}_{du}^{2}\frac{(\phi-1)}{\phi}-(\phi+1)(\phi-1)=0$$

$$\phi^2+\phi-\mathbf{2Fr}_{du}^{2}=0$$

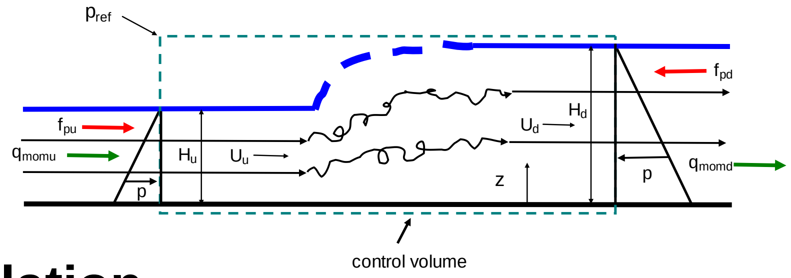
RESULT



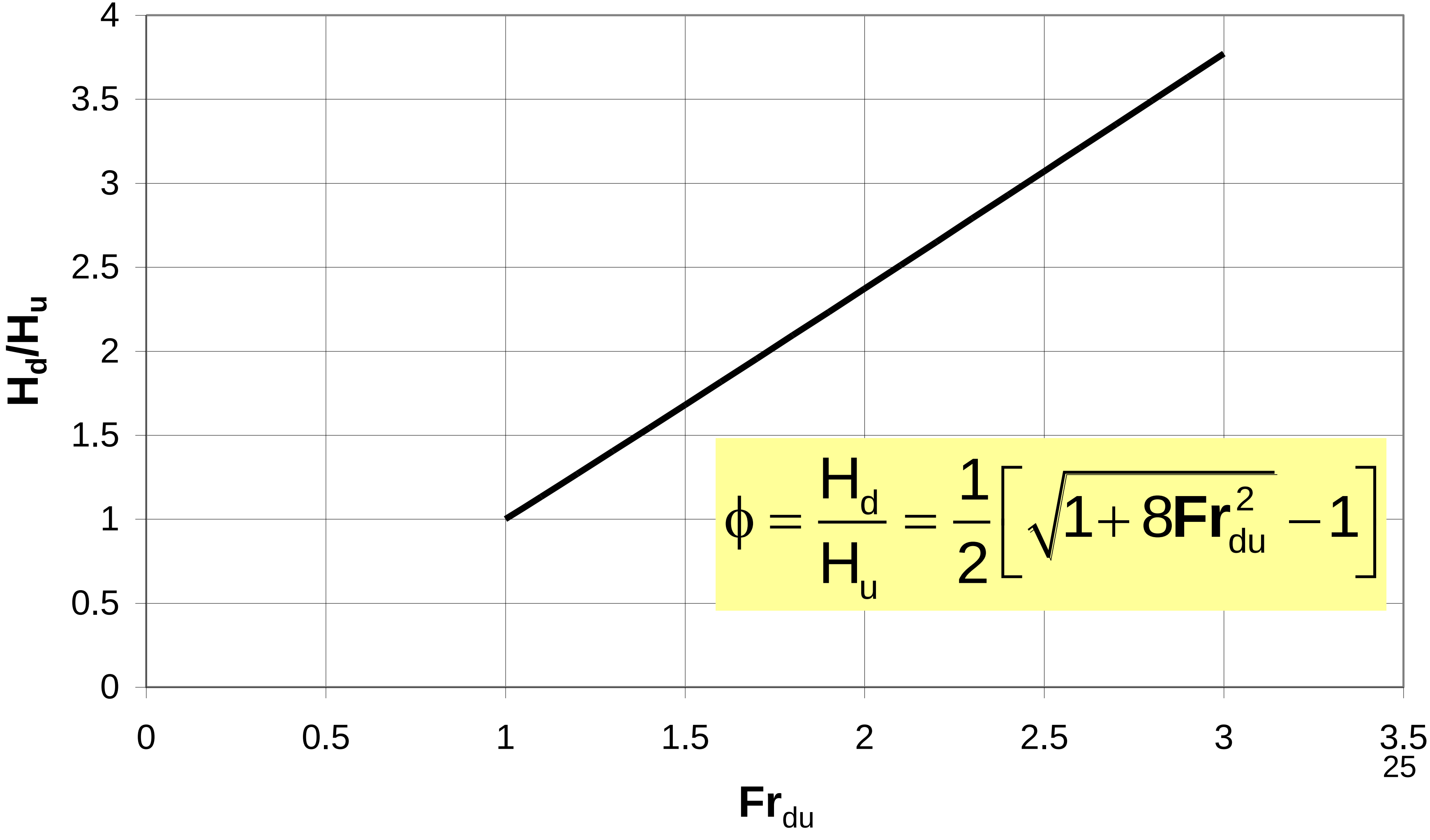
$$\phi = \frac{H_d}{H_u} = \frac{1}{2}\left[\sqrt{1 + 8\mathbf{Fr}_{du}^2} - 1\right]$$

This is known as the conjugate depth relation.

RESULT



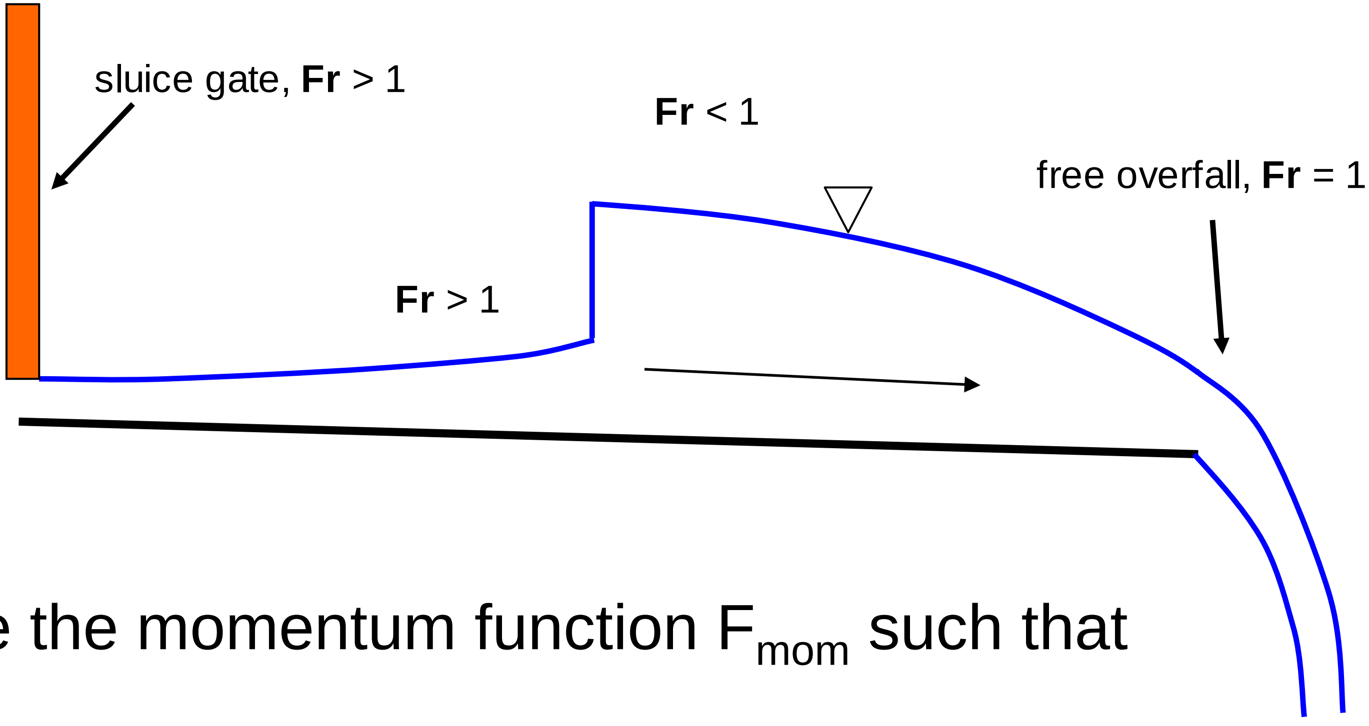
Conjugate Depth Relation



$$\phi = \frac{H_d}{H_u} = \frac{1}{2}\left[\sqrt{1+8\mathbf{Fr}_{du}^2} - 1\right]$$

ADD MATERIAL ABOUT JUMP SIGNAL!
AND CONTINUE WITH BORE!

SLUICE GATE TO FREE OVERFALL



Define the momentum function F_{mom} such that

$$F_{mom}(H) = \frac{1}{2} g H^2 + \frac{q^2}{H}$$

Then the jump occurs where

$$\left[F_{mom}(H)\right]_{left} = \left[F_{mom}(H)\right]_{right}$$

The fact that $H_{left} = H_u \neq H_d = H_{right}$ at the jump defines a shock

SQUARE OF FROUDE NUMBER AS A RATIO OF FORCES

$\mathbf{Fr}^2$ ~ (inertial force)/(gravitational force)

inertial force/width ~ momentum discharge/width ~ $\rho U^2 H$

gravitational force/width ~ (1/2)gH^2

$$\mathbf{Fr}^2 \sim \frac{\rho U^2 H}{\frac{1}{2}\rho g H^2} \sim \frac{U^2}{gH}$$

Here "~" means "scales as", not "equals".

MIGRATING BORES AND THE SHALLOW WATER WAVE SPEED

A **hydraulic jump** is a **bore** that has stabilized and no longer migrates.

Tidal bore, Bay of Fundy, Moncton, Canada

MIGRATING BORES AND THE SHALLOW WATER WAVE SPEED

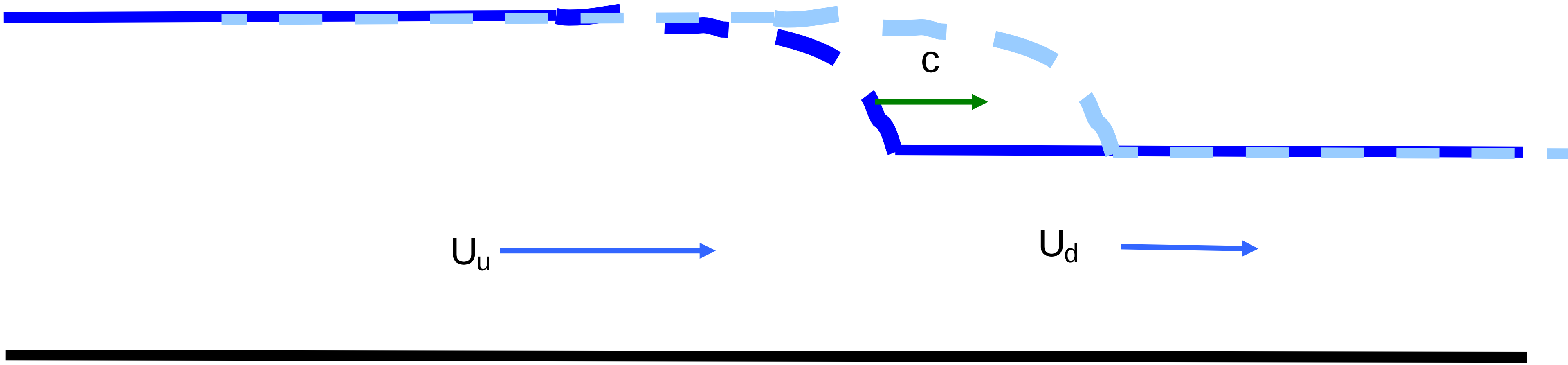
Bore of the Qiantang River, China

Pororoca Bore, Amazon River

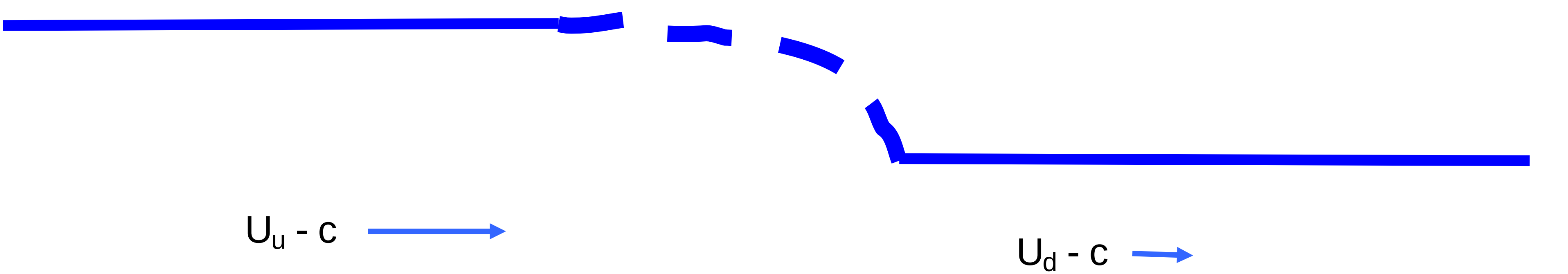
http://www.youtube.com/watch?v=2VMI8EVdQBo

ANALYSIS FOR A BORE

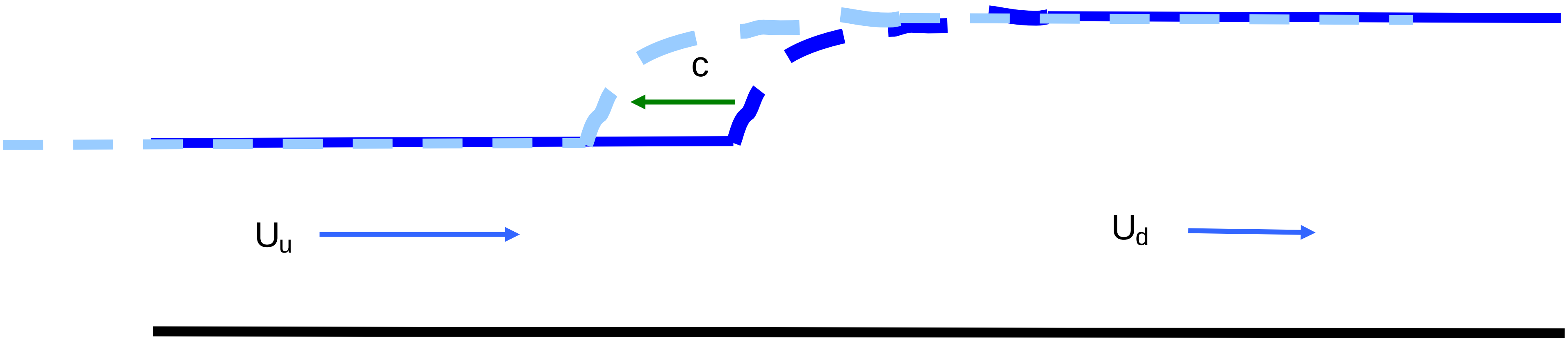
The bore migrates with speed c



The flow becomes **steady** relative to a coordinate system moving with speed c.



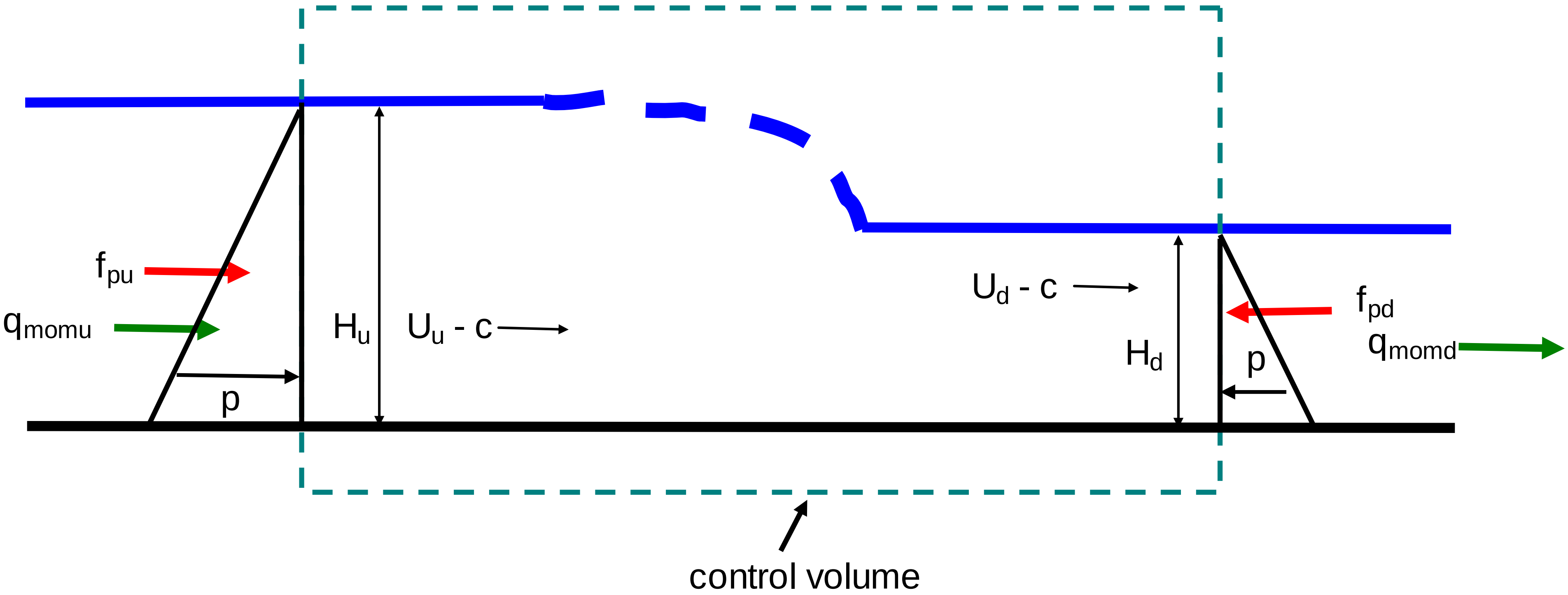
THE ANALYSIS ALSO WORKS IN THE OTHER DIRECTION



The case c = 0 corresponds to a hydraulic jump

CONTROL VOLUME

$q = (U\text{-}c)H$ $\quad q_{mass} = \rho(U\text{-}c)H$ $\quad q_{mom} = \rho(U\text{-}c)^2H$



Mass balance

$$0 = \rho(U_u - c)H_u - \rho(U_d - c)H_d$$

Momentum balance

$$0 = \frac{1}{2}\rho g H_u^2 - \frac{1}{2}\rho g H_d^2 + \rho(U_u - c)^2 H_u - \rho(U_d - c)^2 H_d$$

EQUATION FOR BORE SPEED

$$(U_d - c) = (U_u - c)\frac{H_u}{H_d}$$

$$0 = \frac{1}{2}\rho g H_u^2 - \frac{1}{2}\rho g H_d^2 + \rho(U_u - c)^2 H_u - \rho(U_u - c)^2 \frac{H_u^2}{H_d}$$

$$(U_u - c)^2 H_u\left(\frac{H_u}{H_d} - 1\right) = \frac{1}{2}g(H_u^2 - H_d^2)$$

$$c = U_u \pm \sqrt{\frac{\frac{1}{2}g(H_u^2 - H_d^2)}{H_u\left(\frac{H_u}{H_d} - 1\right)}}$$

LINEARIZED EQUATION FOR BORE SPEED

Let

$$U \equiv \frac{1}{2}(U_u + U_d) \qquad H = \frac{1}{2}(H_u + H_d)$$

$$U_u = U + \frac{1}{2}\Delta U \qquad U_d = U - \frac{1}{2}\Delta U$$

$$H_u = H + \frac{1}{2}\Delta H \qquad H_d = H - \frac{1}{2}\Delta H$$

Limit of small-amplitude bore:

$$\frac{\Delta H}{H} << 1 \qquad \frac{\Delta U}{U} << 1$$

LINEARIZED EQUATION FOR BORE SPEED (contd.)

$$c = U\left(1 + \frac{1}{2}\frac{\Delta U}{U}\right) \pm \sqrt{\frac{\frac{1}{2}g\left\{H^2\left[\left(1+\frac{1}{2}\frac{\Delta H}{H}\right)^2 - \left(1-\frac{1}{2}\frac{\Delta H}{H}\right)^2\right]\right\}}{H\left(1+\frac{1}{2}\frac{\Delta H}{H}\right)\left[\frac{\left(1+\frac{1}{2}\frac{\Delta H}{H}\right)}{\left(1-\frac{1}{2}\frac{\Delta H}{H}\right)} - 1\right]}} \cong$$

$$c = U\left(1 + \frac{1}{2}\frac{\Delta U}{U}\right) \pm \sqrt{\frac{\frac{1}{2}gH^2\left(2\frac{\Delta H}{H}\right)}{H\left(1+\frac{1}{2}\frac{\Delta H}{H}\right)\left[\frac{\Delta H}{H} + o\left(\frac{\Delta H}{H}\right)^2\right]}}$$

$\therefore \quad c \cong U \pm \sqrt{gH}$ Limit of small-amplitude bore

SPEED OF INFINITESIMAL SHALLOW WATER WAVE

$$c \cong U \pm c_{sw}$$

$$c_{sw} = \sqrt{gH}$$

Froude number = flow velocity/shallow water wave speed

$$\mathbf{Fr} = \frac{U}{\sqrt{gH}}$$